Technical Note

RPGs & QRDs

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Technical Note: FYI re RPGs & QRDs

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Acoustic diffusion, the scattering of sound, is essential in architectural spaces where optimum sound distribution and spectral balance are necessary. This includes, but is not limited to, concert halls, rehearsal rooms, recording studios, and critical listening environments.

In older buildings, coffers, ornamentation, sculpted columns, etc., provided surface irregularities of varied shapes and sizes to diffuse sound. Current architectural practice, favoring more streamlined and linear geometry, often produces rooms with insufficient sound diffusion. To provide diffusing elements in the context of modern design trends, newer buildings use splays, zigzags, and curved surfaces. One of the challenges of modern acoustics is to provide adequate sound diffusion within the constraints of modern architectural design.

Reflection phase gratings (RPGs) and quadratic residue diffusers (QRDs) are recent additions to the architectural vocabulary of sound diffusing elements. RPGs and QRDs first appeared in the 1970s and are now being used in many recording studios, rehearsal spaces, and performance halls. With their increasing use, questions often arise about their effectiveness and their historical and theoretical development. Many people also want to know how to design and build QRDs themselves. This monograph is intended to clarify some misconceptions and answer some of the more common questions. It is assumed that the reader has seen QRDs and RPGs and has some familiarity with their general uses.[1, 2]

1. The Players

There are two key people involved in the development of RPGs and QRDs:

Dr. Manfred R. Schroeder was born in Germany in 1926. He holds a Ph.D. in physics from the Universität Göttingen. He was the head of the acoustics research department at Bell Laboratories in Murray Hill, NJ, where he is still active. He is also Professor Emeritus of Physics at the Drittes Physikalisches Institut at the University of Göttingen. As noted below, his work with reflection phase gratings in architectural acoustics dates back to the late 1970's. He is the inventor (so-to-speak) of the quadratic-residue diffuser. To the best of my knowledge, Schroeder's first published description of a QRD and the relevant theory was in 1979. See Ref. [5].

Dr. Peter D'Antonio was born in Brooklyn, NY in 1941. He holds a Ph.D. in chemistry from the Polytechnic Institute of Brooklyn. He has worked at the Naval Research Laboratory in Washington, DC since 1967. He designed and built Underground Sound Recording Studio (his own recording studio); He is a recording engineer of professional caliber and plays a mean bass. His interest in diffusers stemmed from his efforts in the design of recording studio control rooms, one of the first major applications of quadratic-residue diffusers. His passion and expertise in this field grew into a business, RPG Diffusor Systems, Inc., (note the spelling, Diffus***o***r) which now manufactures a variety of devices for acoustic diffusion and sound control. He developed Schroeder's theoretical work on diffusion into a viable business and has made this rather esoteric theory and device available for a variety of architectural applications. See Ref. [7] for his first publication on RPGs.

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1 For those not familiar with QRDs, see the sketches in Addendum 1.

2 I am, by no means, an authority on this subject but I've worked with RPGs both practically and theoretically for several years. If you have any comments or corrections, or if I've left any important questions unanswered, please let me know. I'll issue revisions as needed. — DF.

2. The Basic Terminology

RPG

An RPG is a *reflection phase grating*. This term has its roots in physics, particularly in optics. It was coined by neither Manfred Schroeder nor Peter D'Antonio. Physicists typically refer to these devices by their full technical name, not as RPGs. To the best of my knowledge, the acronym RPG evolved in the audio profession, particularly in control room applications of gratings.

If this term has a familiar ring, it is probably because it is reminiscent of a device you may have seen in high school physics labs: a *diffraction grating*. These gratings have been used in physics since the time of Fraunhoffer (1787-1826) and Young (1773-1829). If you'll recall, a diffraction grating is a plate of glass or metal with tiny parallel and equidistant lines (usually more than 5,000 per inch) scratched into the surface. Shining light on a glass grating causes diffraction (bending) of light waves passing through it. These diffracted waves interfere with each other and produce a pattern of dark and bright areas known as fringes.[1]

The same effect is produced by reflecting light from a metal grating which cannot, obviously, pass light. These metal gratings are, therefore, called *reflection gratings*.

There are many manifestations of gratings in acoustics. Some common examples are: regularly spaced wood strips, corrugated metal deck, lattice work, fluted block, etc. Even the regular rows of audience seats or pews are known to produce diffraction grating effects.

A *phase grating* is essentially the same sort of device but adds an addition level of complexity. In addition to diffracting the incident wave or wave front, a phase grating also introduces some phase variation. To accomplish this, the surface of the grating is designed in such a way that reflections from one element of the grating are out of phase with those from adjacent elements.[2]

Now, think about RPG diffusers you have seen with reference to the terms discussed above. They have regularly spaced wells and separators serving as a standard diffraction grating surface. Sound is reflected from the diffuser surface (it can't pass through the diffuser) so we clearly have a reflection grating. In addition, the different well depths produce phase variation between adjacent grating elements. Putting it all together produces a reflection phase grating.

In architectural acoustics, a reflection phase grating functions as a diffuser. It diffuses or scatters sound that strikes its surface. However, the extent[3] of diffusion produced by different diffusing surfaces is strongly dependent on the nature of the surface. The size, shape, and irregularity of surface convolutions all affect diffusion. There is also (usually[4]) some manifestation of frequency-dependent behavior related to the size and spacing of the diffusing elements.

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1 Incidentally, interference phenomena of the type produced by diffraction gratings are classic demonstrations of the wave-like behavior of light. As you may know, light exhibits both wave-like and particle-like behaviors. Observations of these phenomena led to the development of quantum mechanics.

2 Actually, the scratched-glass grating described above is a phase grating too. However, for our purposes, this simplified explanation will do. See Refs. [1] and [2] or any standard text on optics for a complete explanation.

3 You may note some difficulty in discussing diffusion quantitatively. There are no widely accepted quantitative terms or measures for diffusion in architectural acoustics. There are more precise terms for diffraction (e.g., the energy and direction cosines of diffracted orders) but they are not readily applied to architectural acoustics.

4 There will be no observable frequency-dependent effects if the wavelengths of the incident sound are much larger (or smaller) than the size and spacing of surface perturbations.

It seems that it would be desirable to have a perfect diffuser, i.e., one that scatters sound energy equally in all directions no matter what the frequency content or direction of arrival of the wave. Be careful here! This is another one of those logical-sounding but hard-to-substantiate assumptions that plague architectural acoustics. There is no convincing evidence that a “perfect” diffuser is any more desirable than a “pretty-good” diffuser. On the contrary, there is evidence to suggest that a perfect diffuser is not desirable for many common applications [3], [4].)

QRD

A QRD is a *quadratic-residue diffuser.* It is just one of many possible forms of reflection phase gratings. The term (not the acronym) was first used by Schroeder to describe an optimum acoustic diffuser surface he had devised for use in concert halls [5]. This surface is a reflection phase grating with the phase variation produced by the different depths of adjacent wells. Any similar assembly, i.e., one with equally spaced wells of different depths, will function, to some degree, as a reflection phase grating. However, in the QRD, the well depths are based on a numerical sequence known as the quadratic residue sequence. This numerical sequence has its roots in a branch of mathematics known as elementary number theory.

The term *quadratic residue* may also have a familiar ring to it. (Perhaps you remember quadratic equations from elementary algebra.) The word quadratic tells us that the sequence is derived by use of a second order (squared) term. Recall that a quadratic equation is a second-order equation. (For example: $x^2 + xy + 7 = 0$.) The word *residue* refers to the fact that the sequence is derived from something *left over* after a mathematical operation is performed. In simple division, we call the residue the “remainder.” So you see, the intimidating term “quadratic residue” refers to some basic concepts with which you are already familiar. (The mathematical expression for the quadratic-residue sequence is shown in the next section.)

As purely mathematical concepts, quadratic residues were first formulated and studied by Gauss (1777-1855) and Legendre (1752-1833). Though the quadratic residue sequence was not invented by Schroeder, he was the first to apply it to acoustic diffusing surfaces [5]. Note that Schroeder has also experimented with diffusers using other numerical sequences. (See Addendum 2, at the end of this paper on Primitive Root Diffusers.) He has also applied the quadratic residue sequence to other fields such as communication theory, error correction in digital systems, microwave antennae, etc. [6].

In experiments with various number sequences (by computer simulations), Schroeder and his associates at Göttingen found that a reflection phase grating based on quadratic residue sequences yielded extraordinarily uniform diffusion. It is curious that there is no special reason why this should be true. There are no innate physical or acoustical principles embodied in the quadratic residue sequence that might lead one to believe that this should be the case.

In Schroeder’s earlier theoretical work he experimented with many different number patterns, including random sequences, pseudo-random sequences, and primitive roots. It seems to have been serendipity that, in groping for unusual number patterns, Schroeder stumbled upon one that, when applied to the phase relationships of interfering waves, produced highly uniform diffusion. In more precise language: the discrete Fourier transform of the exponentiated quadratic residue sequence has constant magnitude. This was a rather remarkable and unexpected result.

Schroeder reported his findings in JASA [5] and presented the relevant theoretical background. The associated experimental work was conducted at the University of Göttingen using microwaves—not acoustic waves. As far as I know, most of the experimental work with acoustic waves has been done by D’Antonio [7]. Most of his work has been done using a measurement technique he devised using a TEF™ analyzer.

The term *Schroeder Diffuser* was first used (I believe) by Strube in a JASA article [8] discussing the theory of Schroeder's quadratic-residue diffuser. Strube is an experimentalist (as compared to Schroeder who is, essentially, a theorist) and an associate of Schroeder's at the University of Göttingen.

In view of the commercialization of various diffusers and the trademarking of RPG™ and QRD™ by D'Antonio, and recent product releases by Wenger, I think it is appropriate that we in the acoustical consulting profession acknowledge Schroeder's pioneering work by referring to such diffusers as "Schroeder Diffusers" unless referring specifically to trademarked products.

3. The Mathematical Basis for a Schroeder Diffuser Design

Designing a Schroeder Diffuser is a fairly simple process. All the calculations can be done with a hand calculator. When done in the context of architectural constraints, the design process is typically a trial-and-error procedure. The designer can, with a little experimentation, arrive at dimensions that will satisfy the architectural imperatives as well as the acoustical requirements. Let's begin with a little mathematical groundwork.

Prime Numbers

Generating a quadratic residue sequence calls for a prime number. As you will recall, a prime is a number that is exactly divisible only by 1 and itself. There is an infinite number of primes but there are only a few that are useful for diffuser design because of practical and dimensional limitations. (The largest prime I've seen used in the design of a Schroeder diffuser is 53.)

There are 26 primes less than 100:

1	2	3	5	7	11	13
17	19	23	29	31	37	41
43	47	53	59	61	67	71
73	79	83	89	97		

The Quadratic Residue Sequence

The quadratic residue sequence (let's call it S_{QR}) is simply an ordered set of numbers determined by the following formula:

$$S_{QR}(P) = n^2 \bmod P \qquad (1)$$

Where $n = 1, 2, 3, \ldots P$
P = a prime number

In case you're not familiar with it, the mod (modulo) function is also known as the "remainder" function. Calculating $y \bmod x$ gives the remainder of $y \div x$. For example:

$7 \div 2 = 3$ with a remainder of 1 *or* $7 \bmod 2 = 1$

$27 \div 6 = 4$ with a remainder of 3 *or* $27 \bmod 6 = 3$

Now, let's generate a quadratic residue sequence based on Eq. (1). Let's pick 17 as our prime number. (For the moment, I will dispense with any acoustical or architectural considerations such as frequency range or diffuser size.)

$$S_{QR}(17) = n^2 \bmod 17 \qquad \text{for } n = 1, 2, 3, \ldots 17$$
$$1^2 \bmod 17 = 1$$
$$2^2 \bmod 17 = 4$$
$$3^2 \bmod 17 = 9$$
etc.

When you have finished, you will have the following sequence of numbers:

$$S_{QR}(17) = 1,\ 4,\ 9,\ 16,\ 8,\ 2,\ 15,\ 13,\ 13,\ 15,\ 2,\ 8,\ 16,\ 9,\ 4,\ 1, 0.$$

This entire sequence is called a complete period. Notice that our prime was 17 and the sequence is made up of 17 numbers. Also notice that the sequence is symmetric about the center number (13 in this example) disregarding the final zero. To construct a diffuser of more than one period, simply continue the process again and tag the next sequence of numbers onto the end of this one. This produces a repeating, periodic number sequence.

Figure 1 shows a Schroeder Diffuser consisting of two periods of 17 wells per period. (This figure is based on the quadratic residue sequence derived above with a prime of 17.) The sequence gives the *proportional* relationship of the well depths. The determination of the actual well depths will be shown in the next section.

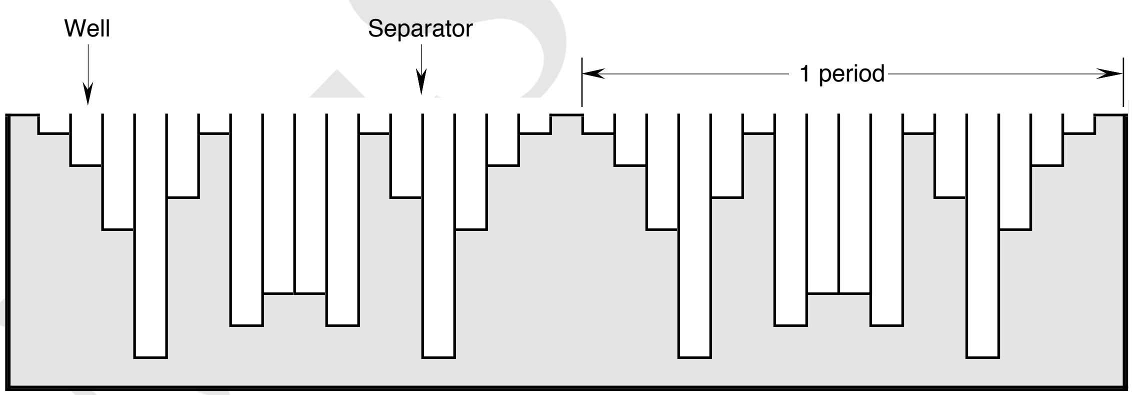

Figure 1. Section through a Schroeder Diffuser. This unit is composed of two adjacent periods based on the prime number 17. Note that the well widths are equal, and the depth variation is symmetric and periodic.

4. Designing a Schroeder Diffuser

You may be wondering how all of this will fit together to allow you to design a Schroeder Diffuser. What about frequency? What prime should be used? You have all the pieces to the puzzle. What you need now is a method for applying this information. The following procedure will work in most situations. As stated above, the most suitable design may require trial-and-error and some compromises between acoustics and architecture.

1. Establish the diffusion frequency range, i.e., the maximum (f_{max}) and minimum (f_{min}) frequencies to be diffused. In general, this range should be as broad as possible. However, the selection of this range will lead directly to the prime number to be used in the design. In particular, the prime number *P* to use for calculations is the nearest prime equal to, or greater than, the frequency ratio:

$$P \geq \frac{f_{max}}{f_{min}} \qquad (2)$$

   For example, if the lowest frequency is 500 Hz and the highest, 8000 Hz, the ratio is 16. The nearest prime (greater than 16) is 17.

   You may find that the selected frequency range results in a hard-to-build design. As will be shown later, the prime number establishes how many wells there will be. But, you should think twice about the cost of construction of a diffuser having, for example, 89 wells. Typically, with too large a frequency range, the number of wells makes construction too difficult or costly, or the well width too small to be practical. As a rule of thumb, start with the largest reasonable frequency range you can. Then, if the design is too complicated, try a smaller range. You will find that, in general, the low frequency is the limiting factor.

   You could approach this differently. Let's say that the low frequency is to be 400 Hz and the prime (the number of wells) is to be 13. Then the highest frequency that will be effectively diffused will be 13 x 400 or 5,200 Hz.

2. The well width is a function of the maximum frequency f_{max}. To provide sufficient diffusion for the highest frequency (shortest wavelength), the well width should be less than one-half this wavelength. The design well width *W* is given by:

$$W \leq \frac{c}{2f_{max}} \qquad (3)$$

   Where c = the speed of sound (1,128 ft/s at 70°F = 13,536 in/s)

3. Now the fun part. The well depths D_{QR} are calculated from the following expression:

$$\mathrm{D}_{\mathrm{QR(n)}} = \frac{c}{2Pf_{min}} S_{QR(n)} = KS_{QR(n)} \qquad (4)$$

   Where:
   - $D_{QR(n)}$ = the depth of the nth well in inches
   - P = the prime number *(from step 1 above)*
   - f_{min} = the minimum diffuser frequency
   - c = the speed of sound in inches
   - $S_{QR(n)}$ = the nth quadratic sequence number
   - K = $\frac{c}{2Pf_{min}}$ *(K is simply a constant)*

An example should clarify any remaining questions:

Step 1. Select a frequency range and calculate the prime *P*.

Let's pick the range 600 Hz to 9,500 Hz. Then

$$f_{min} = 600 \text{ Hz} \quad \text{and} \quad f_{max} = 9{,}500 \text{ Hz}$$

$$P \geq \frac{f_{max}}{f_{min}} = \frac{9500}{600} = 15.8$$ ® **17** *(the nearest prime > 15.8)*

Step 2. Calculate the well width *W*.

$$W \leq \frac{c}{2f_{max}} = \frac{13536 \text{ in/s}}{2 \times 9500 \text{ cycles/s}} = 0.71''$$ » $\frac{11}{16}''$ *(or less)*

Step 3. Calculate the well depths D_{QR}.

Let's start by calculating the quadratic residue sequence for the prime number 17.

$$S_{QR}(17) = n^2 \bmod P \text{ for } \quad n = 1, 2, 3, \ldots 17$$

You should wind up with the following sequence of numbers:

$$S_{QR}(17) = 1, 4, 9, 16, 8, 2, 15, 13, 13, 15, 2, 8, 16, 9, 4, 1, 0.$$

Next calculate the constant K from Eq. 4.

$$K = \frac{c}{2Pf_{min}} = \frac{13536 \text{ in.}}{2 \times 17 \times 600} = 0.6635 \text{ inches}$$

Now, using the quadratic sequence numbers, one at a time, calculate the depths using the following formula:

$$D_{QR(n)} = \frac{c}{2PF_{min}} S_{QR(n)} = KS_{QR(n)}$$

For $S_{QR} = 0$ you should get $D_{QR(n)} = 0$

For $S_{QR} = 1$ you should get $D_{QR(n)} = 0.6635''$ » 0.7 inches

The entire set of the well depths in inches (to the nearest $\frac{1}{10}''$) is:

0.7, 2.7, 6.0, 10.6, 5.3, 1.3, 10.0, 8.6, 8.6, 10.0, 1.3, 5.3, 10.6, 6.0, 2.7, 0.7, 0.0.

— *QED!* —

(Note: If your results don't agree exactly, check for rounding discrepancies. Don't round values until the final operation. Also, a variation in room temperature of only a degree or two will change the speed of sound enough to alter some of these values. Since we can't predict or control the exact temperatures in real rooms, it doesn't make sense to calculate diffuser dimensions down to anything less than 1⁄10 of an inch.)

5. General Information for Building Schroeder Diffusers

Schroeder diffusers are available through RPG Diffuser Systems, Inc. in several standard configurations or in custom form. However, there are often reasons to design and construct one from scratch. The process outlined above provides conservative design parameters for virtually any application. However, for those of you about to actually design or build a diffuser, there may be additional and more practical questions. Hopefully, these are addressed in the following.

What construction materials should be used?

Schroeder diffusers have been constructed from many different materials including wood, particle board, glass, Plexiglas, and metal. For the basic frame, use a material that is easily workable, has the necessary structural strength and dimensional stability, and which meets applicable fire codes. For most uses, particle board is a good choice.

The separators (or ribs) should be as thin as possible. Sheet metal or $^{1}/_{16}$" aluminum is commonly used. Thin wood, Masonite, or Plexiglas can also be used but the thickness should be the minimal. I recommend a maximum of $^{1}/_{4}$". In general, it is not critical whether the thickness of the separator is subtracted from the well width. This will allow some flexibility in the total diffuser width. If there is any need for a preference, I suggest that you reduce the well width to accommodate any space limitations. This will, of course, be limited by construction difficulties. (See further remarks below.)

What are the trade-offs in the design parameters?

Using the calculations presented in Section 4, you can easily come up with several different designs to roughly fulfill the specified frequency limitations. What are the important acoustical factors to consider in the design? [1]

1. **Keep the separators as thin as possible.** There are two reasons: (1) The whole principal of operation of a Schroeder diffuser is the mutual interference between scattered wavelets from <u>all</u> wells. To maximize this interference, it is essential that the wells be as close together as possible. (2) The exposed face of the separators can serve as a specularly reflecting surface when its dimensions approach $^{1}/_{2}$" or more. These secondary specular reflections will reduce the uniformity of the diffuse field you are attempting to produce. You will find further information regarding diffusers with separators of "finite" dimension in Refs. [7] and [9].[2]

2. **Use as many wells as practical.** The more wells, the more diffracted orders; the more diffracted orders, the better the diffusion. For example, at its minimum frequency, a single Schroeder diffuser with 17 wells will scatter sound in broad lobes at 5 discreet angles. A diffuser with 89 wells will scatter in 25 different directions at its minimum frequency [7]. However, an 89-well diffuser is a rather complex and costly beast to construct.

3. **Keep all interior surfaces as rigid, planar, and reflective as possible.** Schroeder diffusers are fairly lossy. Absorption in the wells will tend to further increase the sound absorption. There may be a very good reason to do this but, for now, I would advise against it.

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[1] As you read this, remember that the basic theory for Schroeder diffusers is based on the usual realistic postulates: diffusers made up of an infinite number of periods with separators of infinitesimal thickness. Real-world diffusers never meet these criteria.

[2] To further complicate the issue, Schroeder has stated that separators are not necessary [10]. That may be true for microwaves or waves at zero-degree incidence. However, I do not agree that this applies to ordinary acoustic waves at non-zero-degree incidence. There is strong theoretical support in this regard [7].

What are the absorption coefficients of Schroeder Diffusers?

Naturally, the absorption depends on the construction materials and the specific dimensions. The following data were provided by Peter D'Antonio.

1/1-Octave Band Absorption Coefficients for the RPG™ 734W QRD™

Octave Band Center Frequency (Hz)	125	250	500	1K	2K	4K
Absorption Coefficient	0.23	0.24	0.35	0.23	0.20	0.20

I strongly urge caution in the use of these values, even as general guidelines. These data were obtained with TEF measurements, not according to ASTM standard reverberation room test procedures. That doesn't make them wrong, per se. However, I don't know that anyone has done a formal comparison of data obtained with these two different methods. We don't know if we're comparing apples to apples here. Pete has done some Riverbank measurements on his Abfusors™, but not on his RPGs™ (to the best of my knowledge). I would encourage the undertaking of standardized absorption measurements on Schroeder diffusers and on RPGs™.

How precise must the design dimensions be?

You will notice that in most of the calculations in Section 3, results were rounded to the nearest 1/10". That is a sufficient level of accuracy for real-world applications. In specifications or shop drawings, I recommend that you state dimensions with a tolerance of ± 1/16".

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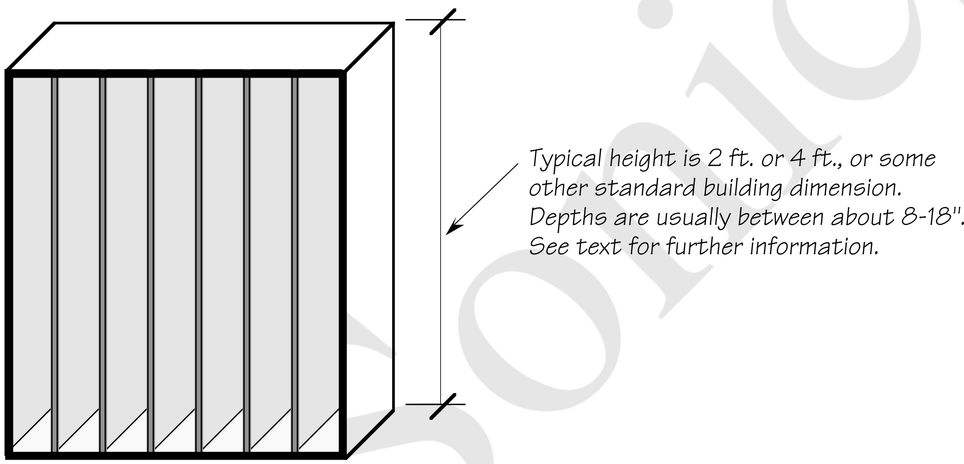
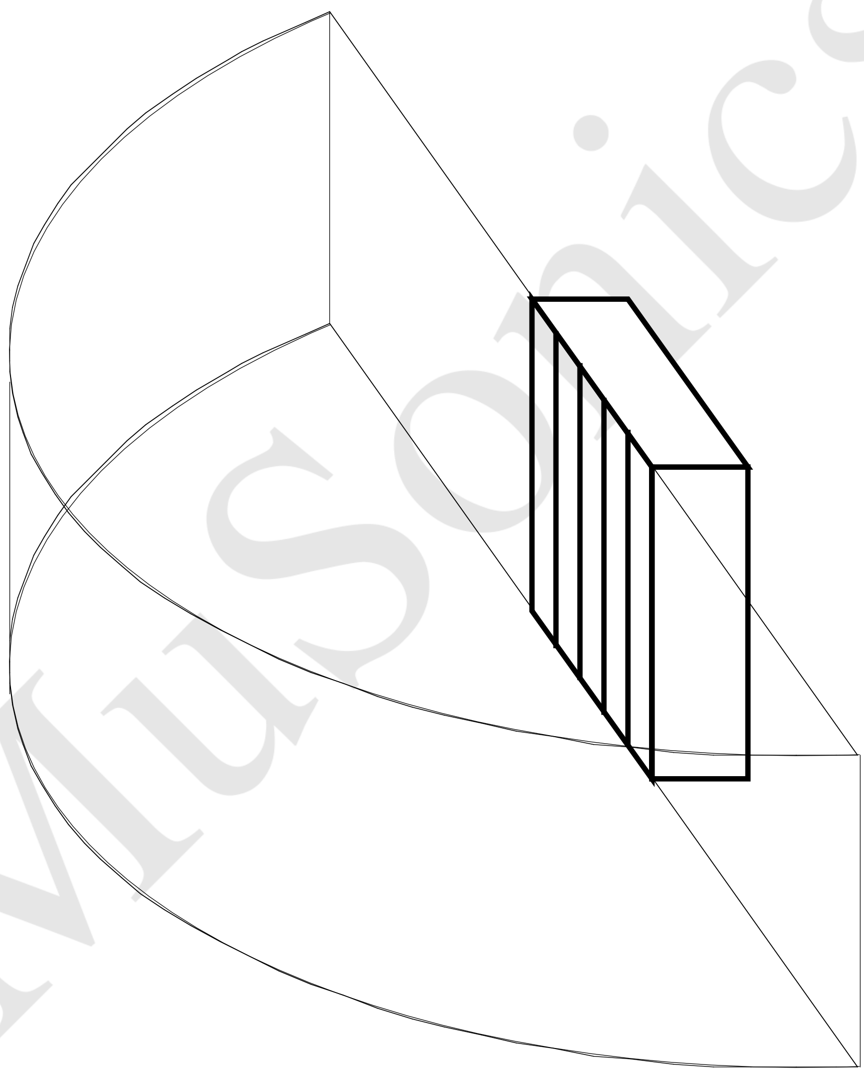
Addendum 1

General Information & Sketches of Typical QRDs

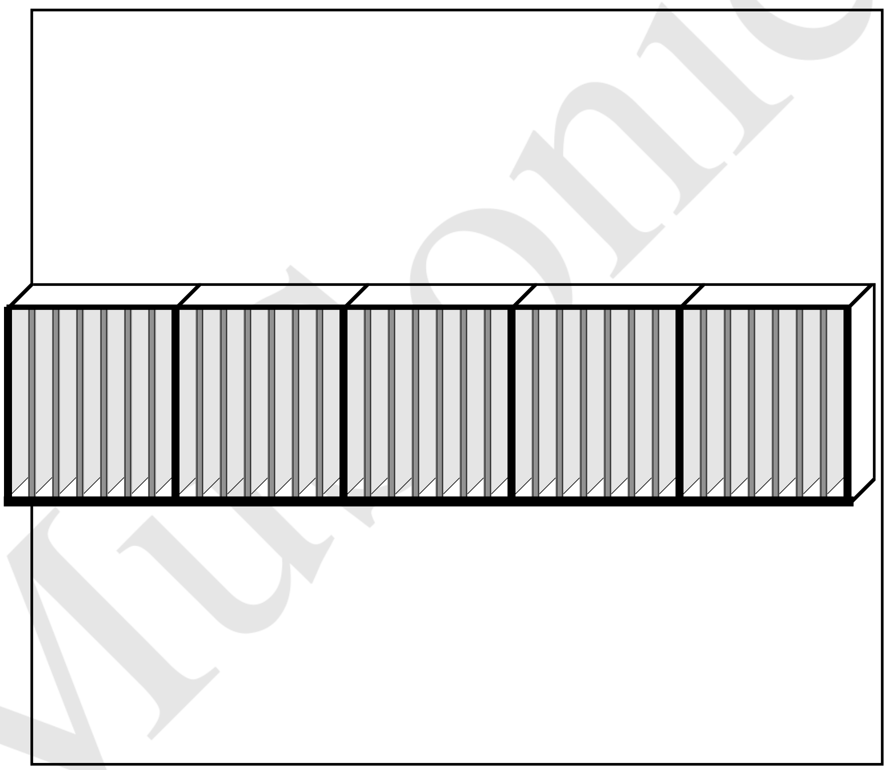
When a sound wave strikes a hard surface, it will be reflected. If that hard surface is also smooth and massive, the sound will be reflected in a specular (mirror-like) way. This sort of reflection leads to echoes and uneven sound distribution. It is usually preferable to have the reflected sound scattered or diffused throughout the room. To accomplish this, hard flat wall surfaces can be covered with sound-scattering devices. One such device is the Schroeder Diffuser (also called QRD or RPG) shown below. These devices look somewhat like shadow boxes or slatted crates.



Diffusion from a Schroeder diffuser is two-dimensional with scattering taking place in a plane perpendicular to the plane of the well separators. This scattering plane is sometimes referred to as a hemidisc. It is important to be aware of this scattering pattern when locating and orienting diffusers.



Typically, several Schroeder diffusers will be applied (side-by-side) over a wall to block specular reflections and produce, instead, a diffuse scattering of sound. Below is a sketch of a wall (roughly 8' tall and 10' wide) with an array of 5 Schroeder Diffusers covering the center portion. This sort of arrangement is often used in recording studio control rooms or small music rehearsal rooms. Note that the well separators are oriented vertically to scatter sound horizontally. In some applications the well separators are oriented horizontally to scatter sound vertically.



Addendum 2

Primitive Root Diffusers

In mid 1992, I was asked to investigate Primitive Root Diffusers. The following summarizes my findings.

Primitive roots are from a sub-specialty of mathematics known as "number theory." Both quadratic residue sequences and primitive root sequences are number-theoretic constructs.

As shown above, the quadratic residue sequence (S_{QR}) is an ordered set of numbers determined by the following formula:

$$S_{QR}(P) = n^2 \bmod P \qquad \text{Where} \quad n = 1, 2, 3, \ldots P, \quad P = \text{a prime number} \qquad \text{(A1)}$$

The primitive root sequence (S_{PR}) is another ordered set of numbers determined by:

$$S_{PR}(P) = 2^n \bmod P \qquad \text{Where} \quad n = 1, 2, 3, \ldots P, \quad P = \text{a prime number} \qquad \text{(A2)}$$

The S_{QR} based on the prime 13 (using Eq. A1) is:

1, 4, 9, 3, 12, 10, 10, 12, 3, 9, 4, 1, 0

The S_{PR} based on the prime 13 (using Eq. A2) is:

2, 4, 8, 3, 6, 12, 11, 9, 5, 10, 7, 1

There are some interesting features and comparisons:

1. The QR sequence is made up of P numbers and is symmetric about the middle. There is one zero and, since there are duplicates of numbers, not all the numbers between 1 and P-1 appear in the sequence.

2. The PR sequence is made up of P-1 numbers and each number from 1 to P-1 appears exactly once: there are no repeated numbers and no zeros. The PR sequence for P =13 is a quasi-random permutation of all the integers from 1 to 12.

Well, so what? PR and QR sequences don't get interesting until we investigate the properties of the Fourier transform of the exponentiated sequences. As shown by Schroeder and D'Antonio [5], [7], for the QR sequence this results in the scattering of incident energy into several discrete orders. The energy scattered to each order is equal. If the incident energy is scattered into 11 different orders, the power reflected in each direction is about 10 dB down from that of the incident wave.

Performing the same operation with the PR sequence also results in the scattering of incident energy into several discrete orders. The energy to each order is equal *except* that there is significantly less energy in the zero-order direction, i.e., in the specular direction. Theory and measurements have shown the power in the zero-order direction to be about 1⁄10 of that to each of the other orders. For the PR sequence, if the incident energy is scattered into 11 different orders, the power reflected in each direction is about 10 dB down from that of the incident wave. But, since the power in the specular reflection is about 1⁄10 of each of the others, it is about 20 dB down from the incident wave.

For the suppression of rear-wall echoes (and using the arbitrary examples above), a QRD should provide a 10-dB reduction of specular reflections; a Primitive Root Diffuser (PRD), a 20-dB reduction. On the surface, that may indicate that the PRD is better at preventing rear-wall echoes [A1], but I'm not convinced that we want that much suppression. For a sense of envelopment in the audience, I surmise that some energy returned from the rear wall is needed including the first reflection of a stage-originating wave striking the rear wall. Recent research supports this and indicates that the "front/back energy ratio" is important in the perception of envelopment [A2].

Also, the PRD suppresses the zero-order reflection for all angles of incidence. This means that a wave striking a rear-wall PRD at non-normal incidence (say after one reflection from a side wall) will have its specular reflection reduced by about 20 dB. Do we really want that?

There are no conclusive answers here—just speculations. However, I haven't seen any documented research on the use of PRDs in real rooms indicating that they are preferable to QRDs, even for rear-wall echo suppression. Also note that Peter D'Antonio had done work with PRDs as early as 1984 and has, to the best of my knowledge, chosen not to promote and market them. Since Pete has been actively researching and marketing every conceivable variation of the diffuser, and since PRDs are no more difficult nor costly to produce than QRDs, I assume he has found no great virtue in the scattering properties of PRDs. I know this is a bit presumptuous of me, but I'll ask Pete the next time I see him.

Postscript

I spoke with Peter D'Antonio in June 1992 about his experience with Primitive Root Diffusers (PRDs). He did quite a bit of theoretical and experimental work with PRDs. He said that both theory and experiment showed that the frequency region of the zero-order suppression was extremely narrow. In fact, his measurements with the TEF showed almost no suppression for broadband signals. He did find significant suppression with CW signals, but this is of little use in room acoustics.

I haven't verified his theoretical findings. However, Pete is such a solid theorist that I don't think this is necessary, though I will take a crack at it eventually. Bottom line: Pete didn't find any real-world benefit in the reputed echo suppression effect of PRDs. He has, therefore, never marketed PRDs and doesn't plan to.

Post Postscript

This monograph was written as an in-house technical note for staff members, so the style is rather breezy. It has not been substantially updated since the early 1990s, so some anecdotal comments are out of date. I believe, however, that the acoustics, math, and physics are accurate.

REFERENCES

A1. M. Schroeder, "Progress in Architectural Acoustics and Artificial Reverberation: Concert Hall Acoustics and Number Theory," J. Audio Eng. Soc. **32**, 194-203 (1984).

A2 M. Morimoto and K. Iida, "A New Physical Measure for Psychological Evaluation of a Sound Field: Front/Back Energy Ratio as a Measure of Envelopment" presented at the 125th ASA meeting, Ottawa, May 1993.